Math Content

Investigate Fractions Tasks

> *There's something weird about fractions. If the numerator, which is the top, is the same, the bigger the denominator the smaller the fraction.*
> *- Jordan, grade 5*

Like Jordan, many students feel that "there's something weird about fractions." This section explores students' challenges with fractions, by examining three tasks and related student responses. Two of these tasks come from the National Assessment of Educational Progress (NAEP).

- If working with a team, we recommend that team members individually solve each of the three tasks shown below, before sharing and discussing your solutions and your ideas about how students might approach each problem. Each task is downloadable.

Problem 1: Estimate the answer to $latex \frac{12}{13} + \frac{7}{8} $. You will not have time to solve the problem using paper and pencil. (Take some notes on your solution method.)

Use the buttons below to view and hide solutions to Problem 1	
Show	Hide

Discussion Questions:

- How did you solve the problem and how might students solve the problem?
- What insights into fractions make quick work of solving this problem?
- Discuss why students chose each of the responses shown above.
- Why do so many students find this problem difficult?

Problem 2: Find two fractions between $latex \frac{1}{2} $ and 1.

Use the buttons below to view and hide solutions to Problem 2	
Show	Hide

Discussion Questions:

- How did you solve the problem and how might students solve the problem?
- What understandings and misunderstandings about fractions might this problem reveal?
- What do the student responses (above) suggest that each student understands and does not understand about fractions?
- Do you notice any differences in the responses of students who used the basal textbook and the students who participated in the "Measure Up"$latex ^{(3)} $ curriculum?

Problem 3: Jim has $latex \frac{3}{4} $ of a yard of string which he wishes to divide into pieces, each $latex \frac{1}{8} $ of a yard long. How many pieces will he have?

Now that you have solved three different fraction tasks and considered student solutions, we suggest that you:

- Briefly summarize your insights into the question “What is difficult for students about fractions?”

Table 1 lists six different aspects of fraction number sense, with examples of student understanding of each.

- Read through Table 1 individually and focus on the connection between the two columns (the type of knowledge and the examples of student difficulty or understanding). Identify any that:
  - Are puzzling or particularly interesting to you.
  - Help you think about any of the student solutions to problems 1 – 3 that you examined.
- Discuss or make notes on these.

Table 1: What’s Hard About Fraction Number Sense?

Type of Understanding or Knowledge	Example of Student Difficulty or Understanding
A Fraction is a Number A Fraction represents an amount, not just pieces (such as 2 of 3 pieces of pizza) or a situation (such as 2 of 3 shirts are red).	When asked to put the fraction $latex \frac{2}{3} $ on a number line, a student said, “you can’t put it on the number line because it’s two pieces out of three pieces, it’s not a number.” [Or “$latex \frac{2}{3} $ is not a number, it’s two numbers.” $latex ^{(1)} $
Partitioning Fractions • A whole can be divided into smaller and smaller equal parts. • The same fractional quantity can be represented by different fractions.	• Difficulty seeing how to divide a whole into *equal* parts. • Difficulty seeing that $latex \frac{1}{2} $ is equal to $latex \frac{2}{4} , \frac{3}{6} , \frac{4}{8} , \frac{5}{10} $… and so on.
The Meaning of the Denominator • The different units (such as $latex \frac{1}{3} $ and $latex \frac{1}{5} $) are different sizes. **The Meaning of the Denominator (cont.)** • The more units as a while is partitioned into, the smaller each one is • $latex \frac{1}{n} $ fits exactly *n* times into the whole.	• Students add $latex \frac{1}{3} + \frac{1}{5} $ and get $latex \frac{2}{8} $ without realizing they are adding two different things (thirds and fifths) sort of like adding apples and hammers. • Students may think” $latex \frac{1}{5} $ is bigger than $latex \frac{1}{4} $ because 5 is bigger than 4. “ • Difficulty seeing that $latex \frac{1}{3} $ fits in the whole 3 times, $latex \frac{1}{4} $ fits in the whole 4 times. Has trouble seeing that $latex \frac{3}{3}, \frac{4}{4} $, etc equal 1.
Knowing What is the Whole • Constructing the whole when given a fractional part. • Keeping track of the whole.	• Difficulty making the whole when you give them a fractional part, e.g.: “This paper is $latex \frac{2}{3} $: show me the whole.” • Sees that the magnitude of a fraction depends on the magnitude of the whole (e.g.,half of a small cookie is not the same as half of a large cookie) • Confusion about whether the two drawings below together represent $latex \frac{3}{8} $ of a pie or $latex \frac{3}{16} $ of a pie.
• Understanding that fraction size is determined by the (multiplicative) relationship between numerator and denominator – not just by the numerator, not just by the denominator, and not by the *difference* between numerator and denominator. • Sees non-unit fraction as an accumulation of unit fractions. [A unit fraction has a numerator of 1; a non-unit fraction has a numerator other than 1.]	• May think $latex \frac{4}{9} $ is bigger than $latex \frac{3}{4} $ because 4 is bigger than 3 (comparing numerators), or $latex \frac{4}{9} $ is bigger than $latex \frac{3}{4} $ because 9 is bigger than 4 (comparing denominators), or $latex \frac{3}{5} $ is the same size as $latex \frac{5}{7} $ because the difference between the top and the bottom in both fractions is 2. • Sees that the quivalent fractions have the same multiplicative relationship between numerator and denominator. In $latex \frac{2}{4} $, $latex \frac{4}{8} $, $latex \frac{3}{6} $, etc. denominator is two times numerator. • Sees $latex \frac{5}{8} $ is made up of $latex 5 \frac{1}{8} $’s or 5 times $latex \frac{1}{8} $, that $latex \frac{9}{8} $ is made up of 9 eights or 9 times $latex \frac{}{} $, etc.

Fractions Can Represent Quantities Greater Than One May be difficult for students who have a strong image of a fraction as a *piece* of something.	• “You can’t have $latex \frac{6}{5} $, because there’s only $latex \frac{5}{5} $ in a whole.”

$latex ^{ (1) } $ Kerslake, D. (1986). Fractions: Children’s strategies and errors. A report of the strategies and errors in Concepts in Secondary Mathematics and Science Project. Windsor, England: NFER-Nelson. Behr, M.J. & Post, T.R. (1992). Teaching rational numbers and decimal concepts. In T.R. Post (Ed.), Teaching mathematics in grades K-8, research-based methods (pp. 201-248). Boston: Allyn and Bacon.

$latex ^{(2)} $ Student work from Work Session presented by Barbara Dougherty and Barbara Fillingim, NCTM Annual Meeting Research Presession, April 21, 2009, Washington D.C., reproduced by permission of first author. Footnote (3) provides more information on the “Measure Up” curriculum.

$latex ^{(3)} $ The “Measure Up” curriculum emphasizes use of units of length, area, and volume to explore basic mathematical ideas such as equivalence. For example, students might compare two lengths by using a third length. Students using this curriculum become very attuned to asking, “What is the unit?” since different units (such as a hexagon and six triangles) might be used to create equivalence.