Math Content

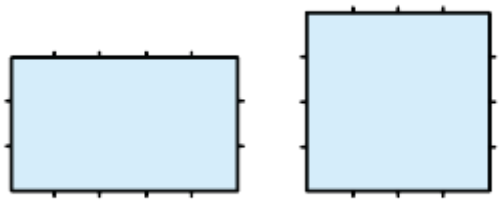
Investigate the Area of Polygons

This section provides two area tasks, followed by information on the development of students' thinking about area. As you study these, fill in items 2-6 of the unit plan template to reflect your thinking.

(1) Area Tasks to Solve and Anticipate Student Thinking

The two Area of Polygons tasks shown in the table below are well-suited to Teaching Through Problem-solving and to developing students' mathematical practices. Depending on what your students already understand, one or both tasks may be useful in your unit plan. If your students do not yet firmly grasp what area is, Problem 1 might be a good way to begin your unit. Once students understand rectangle area and its measurement, Problem 2 provides a good opportunity for you to see whether students can apply their knowledge to an L-shaped figure.

Table AOP1: Area of Polygon Problems

<table>
<tr><td colspan="2">Problem 1
Which of the following shapes, the rectangle or the square, has the larger area and by how much? Think about a way to express how large they are.
  </td></tr>
<tr><td>Problem</td><td>What this problem might build/reveal</td></tr>
<tr><td>1: Compare area of square and rectangle (perimeters of 16 & 15)</td><td>Do students grasp the basic concept of area as 2-D space covered?
Do students distinguish between area and linear measurement?
Do students use direct comparison (overlap) and indirect comparison (using a third object) to compare area?
Do students accurately structure the space within the square and rectangle into units?
Do students accurately use square units to compare area? For example, do they measure without gaps and overlaps of units?
Do students grasp the connection between linear measurement and the number of rows, columns and square units?
Do students understand why and how to use multiplication to find area?</td></tr>
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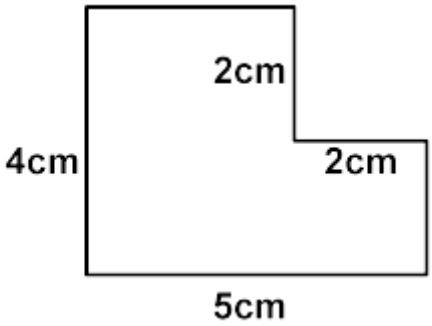
Problem 2

Make the following shape on a geoboard (or dot paper) and find the area. (From "Can You Find the Area?" series of lessons on video by Akihiko Takahashi.)



Problem	What this problem might build/reveal
2: Area of L-shape	Do students see that the L-shape can be decomposed in various ways? That it can be conceived as part of a larger rectangle? Do they see the additive nature of area? Do they accurately structure the space inside the figure into square units? Do they understand what needs to be done in order to use multiplication to find area of the L-shape? Do they write mathematical expressions for the total area?

Solve each task *and anticipate how students will solve it*. Consider the following questions:

- How will students use what they already know to solve this problem?
- What will be common solution methods? What will be common mistakes and stumbling blocks? Why do these mistakes occur?
- What big mathematical ideas would we like students to learn by solving and discussing this task?

Keep track of the student solution strategies you anticipate, recording them in your Lesson Plan Template (Item 8, Flow of the Lesson, under "anticipated student responses").

(2) Investigating the Learning Trajectory for Area of Polygons

According to the Common Core State Standards and related Progressions for Geometry (see especially pages 3-5 and p.16) and Geometric Measurement (see especially pages 16-18), two major understandings to be developed by K-6 students are *composition and decomposition of shapes* and *spatial structuring*. These two understandings are also central to the Japanese elementary mathematics curriculum.

- **Composing and decomposing**. Composing and decomposing figures provides an opportunity for students to learn about area as the "amount of two-dimensional surface that is contained within a plane figure" and to learn that "area is additive, i.e, the area of the union of two regions that overlap only at their boundaries is the sum of their areas" (CCSS progression geometry measurement K-5, 2012, p.4).
- **Spatial structuring.** To measure area, students must mentally structure the space within a figure–for example, mentally structure a rectangle into rows and columns of square iterated units. The CCSS Progression for Geometry (p.4) notes that "Such spatial structuring precedes meaningful mathematical use of the structures, including multiplication and, later area…." and that

"Early composition and decomposition of shape is a foundation for spatial structuring." Students who are still developing an understanding of area may overlap or gap square units as they attempt to measure.

The information above should help you consider how your unit fits within the larger trajectory of learning about area of polygons outlined in the CCSS. You can record your thoughts about this in item 5 of the lesson plan template ("relationship of the lesson to the CCSS"). Use item 6 of the lesson plan template ("research and *kyouzai kenkyuu*") to record any additional insights you have gained into the content and teaching materials through reading and discussion of the standards, progressions and/or Japanese curriculum trajectory.