

Unit Plan for Fractions *[grade and subtopic]*

At *[name of the school]*, *[teacher's name]*

This template is designed to help you record your ideas as you work on Assignments 3 and 4. When you finish, you will have a unit plan draft to share with Network members (which is Assignment 3). Red italicized text briefly describes what goes in each section, and can be deleted. Feel free to edit any text (regular or italicized) to fit your thinking and local situation.

1. Goals for Student Mathematical Practices, and Theory of Action To Achieve Them

Select two of the first five [CCSS mathematical practice standards](#) that you would particularly like to focus on. Note them in the right-hand column, along with any evidence you might expect to see if students are showing these mathematical practices. In the left and middle columns, describe the classroom instruction that you think will help students develop these practices. Feel free to express your ideas in a different sort of diagram, if-then statements, or in some other way that makes sense to you. Notes from the Summer Institute should be useful in recalling your thoughts about the mathematical practices and Teaching Through Problem-Solving instructional elements you would like to work on.

Theory of Action

Classroom Instruction	So Students Learn/Feel	As a Result They Build Mathematical Practices :
<p>Task that requires students to use mathematics to figure out something new (not just to practice something already learned)</p> <p>Emphasizes mathematical <i>practices</i>, not just answer</p> <p>Board shows coherent story of lesson, so students can independently revisit and use it to make sense of mathematical ideas</p> <p>Teacher's questioning focuses on sense-making (Can you write a number sentence to go with this solution? Can someone explain why Pat did this?)</p> <p>Classmates' ideas are audible, visible, selected by teacher to help class build important mathematical ideas</p>	<p>Mathematical practices and prior knowledge help you figure out something new</p> <p>Their own partial ideas and mathematical practices are useful even if they don't immediately give the right answer—for example, writing numbers to go with a physical representation, asking a peer to explain an equation, making an orderly, labeled drawing</p> <p>Classmates may offer important ideas and strategies</p> <p>Math may require effort, revision, comparing different ideas</p>	<p>MP#1 Make sense of problems & persevere in solving them</p> <p>Students</p> <ul style="list-style-type: none"> - write and discuss willingly, to make sense of problem, - are interested in finding & understanding different strategies - connect different representations (verbal, visual, numeric) - use various resources to make sense of problems, such as peers' ideas, prior knowledge, board, etc.



[Return to Step 2 of the Step-by-Step Support for Fractions](#)

Title of the Unit: **Introduction to Fractions**

2. Flow of the Unit (Put a * next to lessons you will plan in detail)

Briefly describe the lessons that are part of your planned unit (whether or not you will actually teach these lessons). Feel free to use the ideas below, or to adapt as needed to reflect your students' prior learning, your local requirements, time devoted to the topic, and so forth. Choose two lessons that you would like to plan in more detail, and asterisk those. (Or come back and choose them later.) Note with a "T" any lessons you will actually teach. The unit plan example below draws in part from a plan developed by Akihiko Takahashi.

Goals of the Unit:

Students will understand the meaning and the representations of fractions in simple cases and appropriately use them.

- To understand that fractions are used to express an amount obtained as a result of equal partitioning and are used to express quantities less than 1
- To understand that a fraction can be considered as a collection of unit fractions
- To understand fraction notation (Formal notation of fractions such as a/b is not introduced here; words are used to describe the fractional part)
- To become aware that a fraction can also be put on a number line like whole numbers
- To become aware that addition and subtraction can also be applied to fractions

Lesson	Title, Learning Activities	Objectives
1	How can we express fractional parts (1)? Find the length of a mystery piece, using an (unruled) meter as a reference. (Unit fractions only)	Students become aware that fractions can be seen in everyday life. Students understand that fractions express an amount obtained as a result of equal partitioning.
2	How can we express fractional parts (2)? Find the length of a mystery piece, using an (unruled) meter as a reference. (Non-unit fractions).	Students understand that a fraction can be seen as a collection of unit fractions. Students learn fraction notation.
3	The size of fractions (1) Create lengths that are fractions of a meter. Order fractional lengths on a number line.	Students become aware that a fraction can be put on a number line. Students will become aware that the size of a fraction depends on the whole.
4	The size of fractions (2) Continue activity, including	Recognize and generate simple equivalent fractions.

	generate equivalent fractions using known unit fractions.	
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3. Goals of Instruction:

In addition to the lesson objectives included above, consider (a) your long-term goals for students' development of mathematical practice and (b) and your goals for learning about TTP.

EXAMPLES:

a. Mathematical practice goals for students:

- i. Students are motivated to construct viable arguments and critique the reasoning of others (CCSS MP3) as they solve a challenging problem.

b. Goals for my learning about teaching through problem-solving (“TTP”):

- i. Facilitate *neriage* discussion that allows students to compare solution methods and develop the big mathematical idea of the lesson (that the size of a fraction depends on the size of the whole).
- ii. Use board organization that helps students understand and connect visual and symbolic representations of fractions.
- iii. Support students to present their thinking in ways that can be seen, heard, and understood by classmates.

4. Relationship of the Unit to the CCSS (Examples) :

Key Related Prior Learning:

2.MD.1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

2.MD.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1,2,...

Learning in This Unit:

3.NF.1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF. 2.a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

3. NF. 3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Future Learning:

3. NF. 3c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

4.NF. 3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

4.NF. 3.a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF. 3.b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$

Background and Rationale

This section provides background and rationale for the flow of the unit and lessons, for example: what students understand currently about this topic, what they struggle with (based on past experience), and how the sequence of lessons in the unit will help them develop the understandings outlined in CCSS. Example:

Students may be familiar with some fractions (such as one-half) but may not realize that fractions are made by splitting a whole into equal parts, and they may not realize that $1/2$ is the quantity that goes twice into the whole, $1/3$ is the quantity that goes three times into the whole, etc. Also, they may not realize that a fraction is a collection of unit fractions. The unit uses mystery strips that are unit fractions of a meter, and as students try to describe these pieces as fractions of a meter, they develop a clear image of, for example, $1/4$ as the fraction that goes into the whole 4 times. Over the lessons of the unit, students deepen their image of fractions by finding and creating non-unit fractions (through iteration of known unit fractions), by ordering unit and non-unit fractions on a number line, and by noticing that a single point on the number line can be described by different fractions.

4. Research and *Kyouzaikenkyuu*

This section describes what you learned from looking at different curricula, reading CCSS-related documents, and learning about teaching through problem-solving. How are you using that information in this lesson?

5. About the Lesson Design

*For the two chosen lessons below, explain why you are designing them as shown. For example, what is your rationale for choosing these activities, and for the particular choice and sequence of student responses you propose for the *neriage*?*

Chosen Lesson #1

Chosen Lesson #2

6. Flow of the Lesson:

Chosen Lesson #1

The sections of this lesson plan are just a guide. “Anticipated student responses,” however, should always be included.

Steps, Learning Activities Teacher’s Questions and Expected Student Reactions	Teacher’s Support/Questioning	Points of Evaluation
<i>This column shows the major events and flow of the lesson.</i>	<i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i>	<i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i>
1. Introduction and Posing the Task <i>This section describes a problem or task as it will be presented to students, as well as any introductory material (e.g., context-setting, remembering prior experiences).</i>		<i>How do we know if students understand the task and are motivated to solve it? What prior knowledge is sparked?</i>
2. Anticipated Student Responses <i>This section describes how students might respond to the task, including incorrect solutions and places where students might get stuck. It can be helpful to tag different responses in some way so you can easily keep track of them.</i>	<i>How will the teacher handle the different student responses, especially incorrect solutions, students who get stuck, or students who finish early? What are the specific questions or comments the teacher will pose in each case?</i>	
3. Neriage: “Kneading” Discussion to Compare Solutions and Draw Out Key Mathematics <i>This section identifies the student solution methods that will be shared and in what order, and the questions that will help students understand and compare the solutions and draw out the key mathematical ideas.</i>	<i>What are the ideas to focus on during the discussion? Which student responses are essential to the discussion, and in what sequence will they be presented? What will be done if essential ideas don’t emerge from students? What will students notice? What questions and teacher moves will help students notice, explain, understand?</i>	<i>What will indicate that students are benefiting from the discussion?</i>
<i>(If needed, repeat 2 & 3 for additional tasks.)</i>		

<p>4. Summing up How will the teacher and/or students summarize the main ideas of the lesson? An assessment activity may also be given.</p>	<p>What prompts for discussion or journal-writing will help students summarize what they learned? Assessment activity, if any.</p>	<p>What comments or work will reveal student thinking and any changes in it?</p>
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Chosen Lesson #2

The sections of this lesson plan are just a guide. “Anticipated student responses,” however, should always be included.

<p>Steps, Learning Activities Teacher’s Questions and Expected Student Reactions</p>	<p>Teacher’s Support/Questioning</p>	<p>Points of Evaluation</p>
<p><i>This column shows the major events and flow of the lesson.</i></p>	<p><i>This column shows additional moves, questions, or statements that the teacher may need to make to help students.</i></p>	<p><i>This column identifies what the teacher should look for to determine whether to proceed, and what observers should look for to determine the effectiveness of the lesson.</i></p>
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	<i>any.</i>	<i>it?</i>
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7. Board Plan

In this section provide a diagram for one of the lessons you plan in detail, showing how work on the blackboard will be organized to support the lesson flow above. A board plan template and examples are available (board plans, under Key Resources at right of TTP site).